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SEMIGROUPS APPROXIMATION WITH RESPECT TO SOME AD HOC PREDICATES[1]

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The problem of semigroups approximation with respect to various predicates has been investigated by many scientists. Some necessary and sufficient conditions for the semigroups approximation with respect to such predicates as "equality", "membership of an element to a subsemigroup", "regular conjugation relation", "Green ratio of L-, R-, H- and D-equivalence", "membership of an element to a monogenic subsemigroup", etc. were obtained. However, there were practically no results on the conditions of approximation with respect to the predicate of membership of an element to a subgroup of a given semigroup. The paper presents the necessary and sufficient condition for approximation with respect to this predicate. We constructed a special semigroup acting the role of a minimal approximation semigroup for many predicates. This semigroup has neither identity nor additive identity. It contains an infinite number of idempotents, and the presence of each idempotent is mandatory. By this semigroup, we have successfully solved the problem of approximation with respect to the predicate of membership of an element to a subsemigroup. A class of semigroups is also described, for which it is the minimal approximation semigroup. We obtained a criterion for the approximation of a semigroup with respect to the Green H-equivalence. The problem of algebraic systems approximating with respect to a predicate consists of three components: a set of algebraic systems (groups, semigroups, etc.); set of predicates; set of functions (homomorphisms, continuous mappings, etc.). The change of one of these components determines a new line of research.

Keywords: *semigroups approximation, approximation with respect to the predicate, minimal semigroup of approximation.*

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A common concept of approximation of the algebraic system is represented by the Russian academician A.I. Mal'tsev [1, pp. 450–462]. In the article, Mal'tsev demonstrates a connection between a finite approximation of the algebraic system with respect to a given predicate and a problem of its solvability in the system. A notion of a finitely approximable semigroup is also mentioned with some results on the semigroup approximation.

The problem of semigroup approximation with respect to various predicates has been investigated intensively during the past decades by Professor M.M. Lesokhin [2] and his students [3–8]. They have found necessary and sufficient conditions for semigroups approximation with respect to many important predicates, such as "equality", "membership of an element to a subsemigroup", "regular conjugation relation", "Green ratio of L-, R-, H- and D-equivalence", "membership of an element to a monogenic subsemigroup", etc.

A lot of research are dedicated to the semigroup approximation with respect to various predicates of important semigroup classes (a class of separative and commutative semigroups, a class of commutative, separative and periodic semigroups, a class of commutative, regular and periodic semigroups, a class of inverse and regular semigroups), but there is no result on the approximation with respect to the predicate of membership of an element to a subgroup. In this article, we present a necessary and sufficient condition for approximation with respect to this predicate.

The problem of finding minimal approximation semigroup was proposed by M.M. Lesokhin [9, 10]. We consider the problem of approximation of semigroups with respect to different predicates. Given a class of semigroups, we intend to find out a minimal approximation semigroup. For some predicates of approximation, the presence of an identity and a zero element is required, especially for a predicate of membership of an element to a subsemigroup, which not only requires the presence of an identity element, but also an external attached identity. We have created a special semigroup, which acts as a minimal approximation semigroup for many predicates. It has no not only identity, but also a zero element. It contains an infinite number of idempotents and the presence of every idempotent is mandatory. We have successfully solved the problem of the minimum approximation semigroups of the predicate of membership of an element to a subgroup.

Preliminaries

Let A and B be two semigroups, Φ be the set of all homomorphisms from the semigroup A to the semigroup B, P be the predicate that is defined on a set that consists of A, all subsets $\delta(A)$ of A and all images of A and of $\delta(A)$ under the homomorphisms from Φ.

Definition 1. Let Q be the set of all prime numbers. Let G_p $(p \in Q)$ be the quasicyclic group of the type p^∞ with identity e_p and with an operation denoted by $\oplus_p$. Put $C^* = \cup G_p$ $(p \in Q)$. Define in C^* as a multiplication as the following: for all $a_p \in G_p$, $a_q \in G_q$,

$$a_p * a_q = \begin{cases} a_p \oplus_p a_q, & if\ p = q; \\ a_{\max\{p,q\}}, & if\ p \neq q \quad \text{and} \quad \max\{p,q\} > 3; \\ e_5, & if\ p \neq q \quad \text{and} \quad \max\{p,q\} = 3. \end{cases}$$

Direct calculations show that $C^* = (C^*,*)$ is a semigroup, a semilattice of groups G_p, $p \in Q$.

For simplicity, in this article we use $a_p a_q$ instead of $a_p * a_q$.

In this article, we consider Φ as all homomorphisms from the semigroup A to the semigroup C^*.

Definition 2. A semigroup A is said to be approximable by homomorphisms from Φ with respect to P, if for a pair of subsets A_1, A_2 from A such that $P(A_1, A_2)$ is false, there exists $\varphi \in \Phi$ such that $P(\varphi(A_1),\varphi(A_2))$ is also false.

Definition 3. A semigroup B is called a minimal approximable semigroup for a class K with respect to P, if the following three conditions are hold:

(i) Any semigroup $A \in K$ is approximable with respect to P by homomorphisms in B;

(ii) If a semigroup S is approximable with respect to P by homomorphisms in B, then $S \in K$;

(iii) If B_1 is a proper subsemigroup of B, then there exists a semigroup $A \in K$ such that A is not approximable by homomorphism in B_1 with respect to P.

Definition 4. Let A be the semigroup. Consider a binary relation on A as follows: for all $x, y \in A$ $(x, y) \in \eta \Leftrightarrow (x, y \in I) \vee (x, y \notin I)$ for every completely isolated ideal I of the semigroup A.

The binary relation η is the least semilattice congruence on A.

Definition 5. A subsemigroup F of a semigroup A is called a filter of A, if for all $x, y \in A$ $xy \in F$ implies $x, y \in F$.

Denote $N(x)$ as the least filter of a semigroup A, which contains an element x and $N_x = \{y \in A \mid N(x) = N(y)\}$. Then N_x be a η-class, which contains the element x and we have: $N_x N_y = N_{xy}$; $N_{x^2} = N_x$; $N_x N_y = N_y N_x$ ([11], Proposition II.2.9).

Other definitions and terms used in this article can be found in [12, 13].

Approximation of semigroups

Lemma 1. Let G_{e_o} be a subgroup of the semigroup A, and e_o be the identity of G_{e_o}. If N_{e_o} contains e_o, then $G_{e_0} \subseteq N_{e_0}$.

Proof. Assume that $g \in G_{e_0}$ and $g \notin N_{e_0}$.

Let $g \in N_g$, $N_g \neq N_{e_0}$ and $g^{-1} \notin N_{g-1}$.

We have $g = ge_0$, $g^{-1} = g^{-1}e_0$, $gg^{-1} = e_0$, so

$$N_g = N_{ge_0}, N_{g^{-1}} = N_{g^{-1}e_0}; N_{gg^{-1}} = N_{e_o}.$$

Next $g = ge_0 = ggg^{-1}$, hence

$$N_g = N_{ge_0} = N_{ggg^{-1}} = N_g N_g N_{g^{-1}} = N_g N_{g^{-1}} = N_{e_0}.$$

This contradicts with the assumption. Thus, every subgroup of semigroup A completely belongs to one η-class.

Lemma 2. For any N_e and any homomorphism $\varphi: A \to C^*$ an image $\varphi(N_e)$ belongs to one maximal subgroup of the semigroup C^*.

Proof. Assume that $x, y \in N_e \Leftrightarrow x\eta y$ are such that $\varphi(x) \in N_p$, $\varphi(y) \in N_q$ and $p \neq q$.

Suppose that $p > q$. Let us consider a set:

$$J_q = \{t \in C^* \mid e_t e_q \neq e_q\},$$

where e_t and e_g are identities of maximal subgroups, containing t and q, respectively.

J_q is a completely isolated ideal of the semigroup C^*.

Denote $J_A = \{x \in A \mid \exists y \in J_q: \varphi(x) = y\}$.

Since $J_A = \varphi^{-1}(J_q)$ and J_q is a completely isolated ideal, then J_A is a completely isolated ideal of the semigroup A.

We have $\varphi(x) \in J_q$, $\varphi(y) \notin J_q$, so $x \in J_A$ and $y \notin J_A$, where J_A is a completely isolated ideal. It means that $(x, y) \notin \eta$. This result contradicts with the above assumption.

Thus, $p = q$ and $\varphi(N_e)$ belongs to one maximal subgroup of the semigroup C^*.

We are now in the position to prove the main result of this article.

Theorem 1. Assume K is a class of semigroups A satisfying the condition: every class N_e of the semigroup A that contains an idempotent is an abelian group. The semigroup C^* is a minimal approximable semigroup for a class K with respect to the predicate of the possible membership of an element to a subgroup of A.

Proof. We follow the Definition 3.

1. Let $A \in K$. We show that A is approximable in C^* with respect to membership of an element to a subgroup of A.

If the semigroup A does not contain a subgroup, then the conclusion of the theorem is trivial.

Let $a \in A$ and G_{e_o} be a subgroup of A, which contains the identity e_0 and $a \notin G_{e_o}$.

A congruence η on the semigroup A divides this semigroup into η-classes, therefore the element a belongs to some η-class N_a. According to the Lemma 1, we have $G_{e_0} \subseteq N_{e_0}$.

For η-classes N_a and N_{e_o}, there are two cases:

a) $N_a \neq N_{e_0}$.

Since η is a semilattice congruence, then $N_a N_{e_0} \neq N_a$ or $N_a N_{e_0} \neq N_{e_0}$.

Suppose that, $N_a N_{e_0} \neq N_a$ (the second case $N_a N_{e_0} \neq N_{e_0}$ is similar).

Consider a homomorphism $\varphi: A \to C^*$ defined in the following way: for all $x \in A$

$$\varphi(x) = \begin{cases} e_p, & \text{if} \quad N_x N_a = N_a; \\ e_q, & \text{if} \quad N_x N_a \neq N_a, \end{cases}$$

where $p > q > 2$.

We have $\varphi(a) = e_p$, $\varphi(G_{e_o}) = e_q$, so $\varphi(a) \notin \varphi(G_{e_o})$.

b) $N_a = N_{e_0}$.

From the condition of the theorem we have that N_{e_o} is an abelian group, so by [14], Theorem 2, N_{e_o} is approximable by homomorphisms $\psi: N_{e_o} \to C^*$.

We have to extend ψ to all A.

N_{e_o} is a subgroup, so $\psi(N_{e_0})$ is a subgroup of C^*, by the Lemma 2, $\psi(N_{e_0})$ belongs to one maximal subgroup $G_{p_0} \subset C^*$.

We have $\psi(a) \in G_{p_0}, \psi(G_{e_0}) \subseteq G_{p_0}$, and $\psi(a) \notin$ $\notin \psi(G_{e_0})$.

Let us consider a map $\mu: A \to C^*$; for all $x \in A$

$$\mu(x) = \begin{cases} e_p, \text{ if } & N_x N_{e_0} \neq N_{e_0}; \\ \psi(xe_0), \text{ if } & N_x N_{e_0} = N_{e_0}, \end{cases}$$

where $p > p_0$ and $p > 3$.

For all $x \in A$, if $N_x N_{e_o} = N_{e_o}$ then $xe_0 \in N_x N_{e_o} =$ $= N_{e_o}$, so the definition of the map is correct.

We shall demonstrate that μ is a homomorphism.

Let $a, b \in A$, $a \in N_a$, $b \in N_b$. There are only two cases:

a) $N_a N_{e_0} \neq N_{e_0}$. Hence $\mu(a) = e_p$ and $\mu(a)\mu(b) =$ $= e_p$.

If $N_{ab} N_{e_0} = N_{e_0}$ then $N_{e_0} \neq N_a N_{e_0} = N_a N_{ab} N_{e_0} =$ $N_{ab} N_{e_0} = N_{e_0}$. It is not possible.

So $N_{ab} N_{e_0} \neq N_{e_0}$ hence $\mu(ab) = e_p = \mu(a)\mu(b)$.

b) $N_a N_{e_0} = N_{e_0}$ and $N_b N_{e_0} = N_{e_0}$.

If $N_{ab} N_{e_0} \neq N_{e_0}$, then

$$N_{e_0} \neq N_{ab} N_{e_0} = N_a N_b N_{e_0} = N_a N_{e_0} N_b N_{e_0} = N_{e_0} N_{e_0} = N_{e_0}.$$

It also is not possible. So $N_{ab} N_{e_0} = N_{e_0}$.

We have $\mu(ab) = \psi(abe_0) = \psi(a(be_0e_0)) =$ $= \psi((ae_0)(be_0)) = \mu(a)\mu(b)$.

It implies that μ is a homomorphism $A \to C^*$ and $\mu | N_{e_0} = \psi$, hence $\psi(a) \notin \psi(G_{e_o})$.

2. Let A be the semigroup and A is approximable by homomorphisms in C^*. We need to show that $A \in K$.

The semigroup A is approximable with respect to the predicate of the possible membership of an element to a subgroup, so A is approximable with respect to the predicate of the possible membership of an element to a maximal subgroup.

In this case, every η-class N_e, containing an idempotent, is a group. We show that N_e is commutative.

Suppose that there are two elements $a, b \in N_e$, such that $ba \neq ab$.

Hence $a^{-1}b^{-1}ab \neq e \Rightarrow a^{-1}b^{-1}ab \notin \{e\}$, therefore, there is a homomorphism χ, such that $\chi(a^{-1}b^{-1}ab) \notin \chi(\{e\})$.

It is not possible, because of the Lemma 2, $\chi(a^{-1}b^{-1}ab) = e \in \chi(\{e\})$.

We have proved that $ba = ab$, hence N_e is an abelian group.

3. Let C_1^* be a proper subsemigroup of C^*.

We need to find out a semigroup $A \in K$, such that A is not approximable into C_1^*.

Let's consider two cases:

a) C_1^* does not contain all idempotents of C^*.

Suppose that $e_p \notin C_1^*$, hence $G_p \not\subset C_1^*$.

Select a semigroup A as a cyclic group with order p and the identity 1_A.

Certainly, $A \in K$.

For any homomorphism $\varphi: A \to C_1^*$, for all $g \in A$

$$1_{C_1^*} = \varphi(1_A) = \varphi(g^p) = (\varphi(g))^p.$$

Every element of C_1^* (except idempotents) has an order different from p, so for all $g \in A$ $\varphi(g)$ is an idempotent and $\varphi(A)$ contains only one element.

Thus, A is not approximable in C_1^*.

b) C_1^* contains all idempotents of C^*.

As C_1^* is a proper subsemigroup, there is an element $g \in C^* \setminus C_1^*$.

Suppose that $g \in G_{q_0}$ and g have an order $q_0^{k_0}$.

So, C_1^* does not contain cyclic subgroups of orders q_0^h, where $h = k, k_0 + 1, k_0 + 2, \ldots$.

Let us select a semigroup A as a cyclic group order $q_0^{k_0+1}$ with an identity 1_A.

Let $\varphi: A \to C_1^*$.

Let a be the element of A and a has an order divided by $q_0^{k_0-1}$.

Because $\varphi(A)$ is a subgroup of C_1^*, so any $\varphi(A) \subset G_{p_0}$ (for some prime number $p_0 \neq q_0$).

At that time $\varphi(A) = e_{p_0}$, or $\varphi(A) \subset G_{q_0}$ and $\varphi(A)$ has an order less or equal to $q_0^{k_0-1}$. Thus, $\varphi(a) = e_{q_0}$ (because the order of the element a is divisible by $q_0^{k_0-1}$).

Hence, the element a and the subgroup 1_A are found in such a way, that for all φ $\varphi(a) = \varphi(1_A)$.

The Theorem 1 has been completely proved.

Let us denote $A^1a = Aa \cup a$; $aA^1 = aA \cup a$; $a\bar{l}b$ if two elements a and b do not belong to Green L-relation, similarly for $a\bar{r}b$; $a\bar{d}b$; $a\bar{h}b$.

The definitions of Green Relations: L-, R-, D-, H-equivalency are in [13, p. 47]. Note, that for the considered above congruence η the ratio $H \subseteq \eta$ is performed.

Theorem 2. A semigroup A is approximable by homomorphisms from Φ with respect to Green relation H-equivalency if and only if the semigroup A is inverse and completely regular semigroup.

Proof. We prove necessary and sufficient conditions.

1. We prove a necessary condition. We know that if $\forall a, b \in A$ ahb, then adb. We show that, in this case, adb implies ahb. Suppose that there exist two elements $a, b \in A$, such that adb but $a\bar{h}b$.

Because the semigroup A is approximable with respect to h-equivalency, there is a homomorphism $\varphi \in \Phi$, such that $\varphi(a)\bar{h}\,\varphi(b)$.

Since $\varphi(a)$, $\varphi(b) \in C^*$, then $\varphi(a) \in G_{p_a}$ and $\varphi(b) \in G_{p_b}$ for some p_a, p_b.

There are two cases:

a) $p_a = p_b = p_0$. Then $\varphi(a) = \varphi(a)\,e_{p_0} = (\varphi(a) \times \varphi(b^{-1}))\varphi(b)$. Hence $\varphi(a)l\varphi(b)$ and $\varphi(a) = e_{p_0}\,\varphi(a) = \varphi(b)(\varphi(b^{-1})\varphi(a))$.

It follows that $\varphi(a)r\varphi(b)$ and $\varphi(a)h\varphi(b)$. This contradicts with the assumption.

b) $p_a \neq p_b$. Suppose that $p_a > p_b$.

Because of adb, then there exists an element $x \in A$, such that alx and xrb. Hence $A^1a = A^1x$ and $xA^1 = bA^1$, therefore there exist elements c and d from A, such that

$$x = ca \text{ and } b = xd, \text{ hence } b = cad.$$

So we have $\varphi(b) = \varphi(cad) = \varphi(c)\varphi(a)\varphi(d)$.

Because $\varphi(c)\varphi(a)\varphi(d) \in G_{p_0}$, where $p_0 = \max\{p_{\varphi(c)}, p_a, p_b\} \geq p_a > p_b$ and two maximal subgroups of C^* cannot have the common element, hence $\varphi(b) = \varphi(c)\varphi(a)\varphi(d)$ cannot take place.

Thus adb follows ahb, and in A the D-class is coincident with the H-class.

Let us assume ahb, but $ab\bar{h}a$. Then there exists a homomorphism $\chi \in \Phi$, such that $\chi(ab)\bar{h}\chi(a)$.

From adb we derive that $a = x_1bx_2$, $b = y_1ay_2$ for some $x_1, x_2, y_1, y_2 \in A$.

Hence $\chi(a) = \chi(x_1)\chi(b)\chi(x_2)$ and $\chi(b) = \chi(y_1)\chi(a)\chi(y_2)$.

It follows that $\chi(a)$ and $\chi(b)$ must be in the same maximal subgroup of C^*, consequently, $\chi(ab)$ and $\chi(a)$ must be in the same maximal subgroup, and $\chi(ab)h\chi(a)$. This contradicts with the assumption.

Thus $abha$, and by [13, p. 87], a class H_a be a group, and A is completely regular semigroup.

We show that every element of A has a unique inverse element.

Let us suppose that the element a has two inverses b and c. Because H_a is a group, then either $b \in H_a$ or $c \in H_a$. Let us assume that $b \in H_a$. Then $c \notin H_a$ (in the opposite case, a and c are mutual inverse, hence $c = b$). By the condition of the theorem, there exists a homomorphism $\mu \in \Phi$, such that $\mu(c) \notin \mu(H_a)$.

We have that c and a are inverse, then $aca = a$ and $cac = c$.

Hence $\mu(a)\mu(c)\mu(a) = \mu(a)$ and $\mu(c)\mu(a)\mu(c) = \mu(c)$, consequently $\mu(a)$ and $\mu(c)$ must be in one maximal subgroup of C^* and $\mu(a)h\mu(c)$. It also cannot take place.

Thus A is the inverse completely regular semigroup.

2. We prove a sufficient condition. Let A be the inverse completely regular semigroup, then A is a union of not intersecting groups $A = \bigcup_{e \in E} A_e$ and idempotents of A are commutative.

Let us assume that $a, b \in A$ and $a\bar{h}b$. Because any two elements of a subgroup of the semigroup A are h-equivalent, then a and b belong to different maximal subgroups of the semigroup A. Let $a \in A_{e_a}$, $b \in A_{e_b}$. Since $e_a \neq e_b$ then either $e_ae_b \neq e_a$, or $e_ae_b \neq e_b$.

Assume that $e_ae_b \neq e_a$.

Consider a set defined as follows:

$$I_{e_a} = \{e \in E \mid ee_a \neq e_a\}.$$

Let us assume that $e \in I_{e_a}$, that is $ee_a \neq e_a$. Then for any $f \in E$, we have $efe_a \neq e_a$. In fact, if $efe_a = e_a$, then $e_a = efe_a = eefe_a = ee_a \neq e_a$.

Consequently $ef \in I_{e_a}$, that is I_{e_a} is an ideal and $E \setminus I_{e_a}$ is a subsemigroup of the semigroup E, from which I_{e_a} is a completely isolated ideal.

Consider a map τ: $A \to C^*$ defined as follow: for all $x \in A$

$$\tau(x)=\begin{cases} e_q, & \text{if} \quad x \in A_{e_x}, \text{ and } e_x \notin I_{e_a}; \\ e_p, & \text{if} \quad x \in A_{e_x}, \text{ and } e_x \in I_{e_a} \end{cases}$$

for any two simple numbers p, q, where $p > q > 2$.

Because I_{e_a} is a completely isolated ideal, then τ is a homomorphism and $\tau(a) = e_q$, $\tau(b) = e_p$, that is $\tau(a)\overline{h}\tau(b)$.

The Theorem 2 has been completely proved.

Conclusion

The problem of approximation of semigroups consists of three components. The first component is a set of algebraic structures such as groups, finite groups, semigroups, compact semigroups, fields, etc.; the second component is a set of predicates; and the last component is a set of functions such as homomorphisms, continuous characters, continuous bicharacters, etc. Changing one of these components will give us a new direction of research.

The problem of finding a minimal semigroup of approximation appears naturally. If we fix a semigroup B satisfying conditions 1 and 2 of Definition 3, we want to find the smallest among all subsemigroups B satisfying these conditions. In any ordered set a minimal element may not be, or be a several. Similarly, the minimal semigroup of approximation for a given class of semigroups is not unique. For example, for the class of regular commutative periodic semigroups two different minimal semigroups of approximation with respect to the predicate occurrence of the element in the subsemigroup were found [10, p. 76].

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АППРОКСИМАЦИЯ ПОЛУГРУПП ОТНОСИТЕЛЬНО НЕКОТОРЫХ СПЕЦИАЛЬНЫХ ПРЕДИКАТОВ

Проблема аппроксимации полугрупп относительно различных предикатов исследовалась многими учеными. Были найдены необходимые и достаточные условия аппроксимации полугрупп относительно таких предикатов, как «равенство», «принадлежность элемента подполугруппе», «отношение регулярного спряжения», «отношения Грина L-, R-, H- и D-эквивалентности», «принадлежность элемента моногенной подполугруппе» и т. д., однако практически не было результатов об условиях аппроксимации относительно предиката принадлежности элемента подгруппе данной полугруппы. В статье найдено необходимое и достаточное условие для аппроксимации относительно такого предиката. Для этого приведена конструкция специальной полугруппы, которая играет роль минимальной полугруппы аппроксимации для многих предикатов. Данная полугруппа не имеет ни единицы, ни нулевого элемента. Она содержит бесконечное число идемпотентов, и присутствие каждого идемпотента является обязательным. С помощью этой полугруппы успешно решена задача аппроксимации относительно предиката принадлежности элемента подгруппе. Также описан класс полугрупп, для которого она является минимальной полугруппой аппроксимации. Получен критерий аппроксимации полугруппы относительно H-эквивалентности Грина. Отметим, что задача аппроксимации алгебраических систем относительно предиката состоит из трех компонентов: набор алгебраических систем (группы, полугруппы и т. д.); набор предикатов; набор функций (гомоморфизмы, непрерывные отображения и т. д.). Изменение одного из этих компонентов определяет новое направление исследований.

Ключевые слова: *аппроксимация полугрупп, аппроксимация относительно предиката, минимальная полугруппа аппроксимации.*

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